

Trees and Turtles: Modular Abstractions for State Machine Replication Protocols

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Abstract

We present two abstractions for designing modular state machine replication (SMR) protocols: *trees* and *turtles*. A tree captures the set of possible state machine histories, while a turtle represents a subprotocol that tries to find agreement in this tree. We showcase the applicability of these abstractions by constructing crash-tolerant SMR protocols out of abstract tree turtles and providing examples of tree turtle implementations. Tree turtles can also be extended to be made Byzantine fault-tolerant (BFT). The modularity of tree turtles allows a generic approach for adding a leader for liveness. We expect that these abstractions will simplify reasoning and formal verification of SMR protocols as well as facilitate innovation in protocol designs.

1 Introduction

State machine replication (SMR) is a widely-used paradigm in distributed and decentralized services, wherein a set of processors provides an abstraction of a single state machine with an ever-growing history [12]. In the face of possible processor failures and unbounded communication delays, the challenge lies in ensuring that nodes always agree on the history while allowing updates to be made in as timely a manner as possible.

Traditional SMR protocols are usually constructed around the notion of an unbounded sequence of slots. The goal of such a protocol is to fill the slots with values. A typical protocol consists of an unbounded series of rounds where the contents of at most one slot may be decided in a round. Some protocols are only able to fill a single slot (i.e., [1, 5]), thus an unbounded number of instances of the protocol must be used to implement SMR. Higher throughput is achieved by running multiple instances in parallel, either independently [4, 9] or in a pipelined fashion [8, 14], or by putting a batch of values in each slot.

We present here two abstractions that break this slot-by-slot paradigm: *trees* and *turtles*. Referring to a sequence of values as a *chain*, the set of such chains forms a tree under the *is-a-prefix-of* relation. As was proposed in [10], we generalize the slot-based scheme for constructing SMR protocols to allow entire chains to be decided at once, extending the state machine history down a path through the tree. We then generalize the notion of a round to an abstract subprotocol which can be used to decide one chain. We refer to

such protocols as turtles because they are stacked *in infinitum* to construct an SMR protocol. Taken together, the result is protocols called *tree turtles*.

Using chains requires our subprotocols to form a consensus out of a set with richer algebraic structures than the traditional set of singular values. In this case, the algebraic structure we use is the meet-semilattice formed by the ancestor relation between the nodes of the tree as the partial order and the lowest common ancestor of a set of nodes as the meet operator. This structure is utilized in the tree turtle protocols and their proofs of correctness that we present in this paper.

We expect that our abstractions can lead to various advantages over traditional SMR approaches. Proposing chains allows processors to specify preferred orderings of values in the state machine history. Reasoning about SMR protocols is made simpler with tree turtles because the never-terminating execution is factored out; this can lead to more reusable proofs in both an informal and formal setting. Tree turtles themselves have simple proofs when compared to existing SMR protocols. The modularity of tree turtles also enables the design of heterogeneous protocols that dynamically adapt to their workloads or operating conditions.

For liveness, we show how to compose our turtle abstraction with a *leader* which attempts to eliminate contention between processors. Different from traditional approaches, our protocol does not require a non-faulty leader to make progress under favorable conditions. We also demonstrate that tree turtles can be made Byzantine fault-tolerant (BFT) by adding an abstraction called *evidence* which allows protocols to use additional techniques for restricting Byzantine behavior (such as digital signatures).

2 Trees and Turtles

Put simply, the goal of SMR is to allow a set of processors agree on an ever-growing sequence of values in a fault-tolerant manner. Rather than focusing on the individual values in a sequence, we will consider how an agreement can be formed on an entire sequence, or a *chain*, at once. Doing so utilizes the tree structure present in sets of chains.

Consider a processor p that believes the state machine history is represented by a chain c . If another processor p' believes the state machine is represented by a different chain

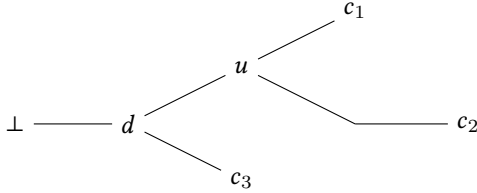


Figure 1. Illustrating the tree structure formed by trees and the is-a-prefix-of relation. Chain u is the longest common prefix of chains c_1 and c_2 . Chain d is the longest common prefix of chains c_1 , c_2 , and c_3 .

c' , then p and p' should be able to agree on a common history. Beyond the simplest case where $c = c'$, consider whether one chain is a *prefix* of the other. If this is the case, the processor who proposed the shorter chain could later “catch up” by extending its chain to the longer chain, without needing to modify the earlier state machine history.

If p and p' attempt to establish agreement on an extension of the state machine history by proposing chains c and c' to each other, then we could consider that p and p' agree on the *longest common prefix* of c and c' . This is the (possibly empty) longest identical subsequence that can be found starting from the beginning of each chain. The longest common prefix can readily be generalized to any number of chains; Figure 1 depicts a tree formed by three chains where the longest common prefixes are ancestor nodes.

Trees¹, then, can be viewed as fundamental to state machine replication: processors continually propose new chains to each other in order to keep extending the longest agreed-upon path through the tree. Table 1 summarizes the notation which we will use for chains in this paper.

Table 1. Summary of notation for chains.

\perp	the empty chain
$c \leq c'$	c is a <i>prefix</i> of c' (or, c' is an <i>extension</i> of c)
$c \simeq c'$	$c \leq c'$ or $c' \leq c$ (we say that c and c' <i>agree</i>)
$c \sqcap c'$	the longest common prefix of c and c'

Now we will turn our attention to the structure of an SMR protocol. To perform SMR, processors alternate between proposing new extensions to the longest agreed-upon chain, collecting proposals from other processors, and deciding what the new longest agreed-upon chain is. This process repeats forever to ensure that the longest agreed-upon chain is ever-growing.

Thus, we may extract a natural building block from this structure: a subprotocol in which processors propose and

¹In addition to being like trees in a loose conceptual sense, chains, taken with the partial order \leq , satisfy the set-theoretic definition of a tree. This is because for each chain c , the set of its prefixes $\{c' \mid c' \leq c\}$ is totally ordered by \leq . As mentioned in the introduction, it is possible to generalize trees even further to a semilattice.

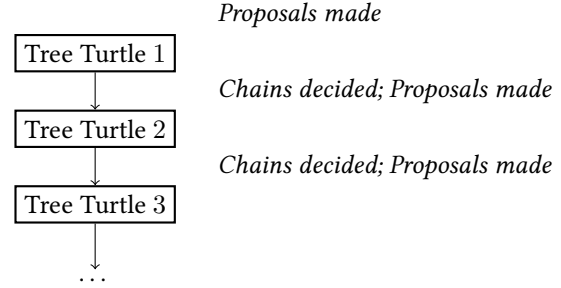


Figure 2. Tree turtle protocols are stacked to implement a state machine replication protocol.

subsequently decide on a single chain. While many subprotocol implementations are possible, we establish a common specification for the properties they should have. Once we have one (or many) such subprotocols, constructing a SMR protocol is simple: we stack these subprotocols on top of each other in an unending sequence. Owing to this infinite repetition, and inspired by the saying “*turtles all the way down*,” we refer to our building blocks of SMR protocols as *turtles*. Since our protocols combine the ideas of trees and turtles, we will call them *tree turtles* (Figure 2).

Tree turtles reduce the problem of solving SMR into making a single proposal and decision. The specification for tree turtles requires them to always terminate (Section 5), while SMR requires that the protocol goes on forever (Section 4). The tree turtle implementations are encapsulated from the correctness of the SMR protocol proved in Section 6 making those proofs reusable across implementations. This also readily demonstrates the potential of heterogeneous SMR protocols composed from multiple types of tree turtles.

3 System Model

We will denote the set of processors participating in a protocol as \mathcal{P} . Processors can communicate with all other processors by sending messages over a network.

Failures. Each processor p can either be correct or faulty. Faulty processors may fail by crashing, at which point they may stop executing indefinitely. Which processors are faulty is not known *a priori*.

Network. The network is assumed to be *reliable* in the following sense: if a processor p sends a message to a correct processor q , then q eventually receives that message. We also assume that the network doesn’t forge or garble messages, meaning that if a processor q receives a message from a processor p , then p actually sent that message.

Asynchrony. We assume that there are no upper bounds on the difference between processor speeds or network latency. This means that processors are not able to distinguish between faulty processors and correct processors that are simply slow or whose messages have yet to be delivered.

Quorums. A processor can never expect to receive a message from all other processors due to the possibility that some have crashed. To address this, a processor instead waits for a *quorum* which is a subset of the processors in \mathcal{P} . A *quorum system* Q is a set of quorums ($Q \subseteq 2^{\mathcal{P}}$) which satisfies the following:

- We assume that any quorum system Q contains a special quorum, which we will denote Q^* , that consists entirely of correct processors.
- We say that a quorum system satisfies *k-intersection* if any set of k quorums have at least one processor in common [7].

Appendix A shows how k -intersecting quorum systems can be implemented using threshold quorums if $|\mathcal{P}| > k \cdot f$, where f is the maximum number of faulty processors.

4 State Machine Replication Specification

Below we give the requirements for a SMR protocol, in which processors in \mathcal{P} alternate *proposing* and *deciding* chains.

Definition 4.1 (State Machine Replication Specification).

- *SMR-Agreement*: if a processor p decides chain d and a processor p' decides chain d' , then $d \simeq d'$ (even if $p = p'$).
- *SMR-Validity*: if a processor decides chain d , then some processor proposed a chain c such that $d \leq c$.
- *SMR-Relay*: if a correct processor decides chain d , then eventually all correct processors decide d or an extension of d .
- *SMR-Monotonicity*: if a processor decides chain d and later another chain d' , then $d \leq d'$.
- *SMR-Progress*: if a correct processor proposes a chain that it has not decided before, then that processor eventually decides a chain it has not decided before.

Note that it is impossible for a protocol to satisfy all properties if the system is asynchronous [6]. For our tree turtle protocols in Section 7, we will focus on the first four properties. We will return to SMR-Progress in Section 8 when we introduce synchrony assumptions.

5 Tree Turtle Specification

A tree turtle is a fault-tolerant protocol executed by the processors in \mathcal{P} . Processors construct an input and produce an output for each tree turtle. An input to a tree turtle is a chain c , which we will denote in brackets $\langle c \rangle$. An output of a tree turtle is a pair of chains $\langle d, u \rangle$. Since tree turtles are subprotocols, we do not assume here that all processors—including non-crashed ones—will execute a given tree turtle. We show in Section 6 that in our construction, all correct processors will in fact execute each tree turtle.

A tree turtle must satisfy the following properties:

Definition 5.1 (Tree Turtle Specification).

- *Turtle-Termination*: if each correct processor constructs an input, then eventually each correct processor produces an output.
- *Turtle-Agreement*: for any two outputs $\langle d, u \rangle$ and $\langle d', u' \rangle$, $d \leq u'$ and $d' \leq u$.
- *Turtle-Unanimity*: for any chain w , if $w \leq c$ for all inputs $\langle c \rangle$, then $w \leq d$ for all outputs $\langle d, u \rangle$.
- *Turtle-Validity*: if some processor produces an output $\langle d, u \rangle$, then some processor must have produced an input $\langle c \rangle$ such that $u \leq c$.

Turtle-Agreement ensures agreement between any two outputs of a turtle: note that if both $d \leq u$ and $d' \leq u$, then either $d \leq d'$ or $d' \leq d$. Further, the case where $\langle d, u \rangle = \langle d', u' \rangle$ implies that $d \leq u$ for each output. Turtle-Unanimity ensures that if there is an agreement in the inputs (i.e., proposals) to a turtle, then that agreement is reflected in the outputs. Turtle-Validity means that each output of a turtle be a prefix of an input to that turtle. Unlike SMR protocols, we are able to guarantee the liveness properties for tree turtle protocols (Turtle-Termination).

6 Tree Turtles All The Way Down

Tree turtle protocols can be composed with each other to implement a protocol satisfying SMR-Agreement, SMR-Validity, SMR-Relay, and SMR-Monotonicity.

The construction works as follows. All processors in \mathcal{P} are configured with instructions to execute the same unbounded sequence of tree turtles numbered $1, 2, 3, \dots$, and so on. For convenience, we will extend the tree turtle inputs and outputs $\langle i, c \rangle$ and $\langle i, d, u \rangle$ to now include the tree turtle number i . We assume that each processor is initialized with the tuple $\langle 0, \perp, \perp \rangle$ as the output for the non-existent tree turtle 0. Then the processors begin executing the tree turtles in succession. If a processor produces tree turtle output $\langle i, d, u \rangle$, that processor decides the chain d . It then proposes a new chain c , selecting c such that $u \leq c$, and constructs an input $\langle i + 1, c \rangle$ for the next tree turtle $i + 1$.

By choice of $u \leq c$, this construction ensures the following:

Lemma 6.1. *If a processor produces the output $\langle i, d, u \rangle$ for tree turtle i , then for all inputs to subsequent tree turtles $\langle j, c \rangle$ where $i < j$, it must be that $d \leq c$.*

Proof. Suppose $\langle j, c \rangle$ is an input to tree turtle j made by a processor p . Proceed by induction on $j - i$. If $j = i + 1$, then by the above construction, p must have produced $\langle i, d, u \rangle$ as the output of tree turtle i for some chain $u \leq c$. By Turtle-Agreement, we also know that $d \leq u$ which implies $d \leq c$. In the inductive case, we again know that p must have produced an output $\langle j - 1, d', u' \rangle$ with $d' \leq u' \leq c$ in the previous tree turtle. The inductive hypothesis gives us that $d \leq c'$ for all inputs to tree turtle $j - 1$ $\langle j - 1, c' \rangle$. Thus,

the condition for Turtle-Unanimity is satisfied for tree turtle $j - 1$, and so it must be that $d \leq d' \leq c$. \square

Further, we can use an inductive argument to see that each correct processor will eventually complete each tree turtle.

Lemma 6.2. *Every correct processor eventually produces an output for each tree turtle.*

Proof. Trivially, each non-crashed processor constructs an input to tree turtle 1. Thus, Turtle-Termination ensures that the base case of the induction is satisfied. In the inductive case, each non-crashed processor will use its output of tree turtle i to construct an input to tree turtle $i + 1$, and so an analogous argument applies. \square

Now we proceed to the main proof:

Theorem 6.3. *The composition of tree turtles implements a protocol satisfying SMR-Agreement, SMR-Validity, SMR-Relay, and SMR-Monotonicity.*

SMR-Agreement. Suppose that two processors p and p' decide chains based on their outputs $\langle i, d, u \rangle$ and $\langle j, d', u' \rangle$, respectively. First, suppose that $i = j$. By Turtle-Agreement, $d \leq u'$, and we also know that $d' \leq u'$. Then, since d and d' are prefixes of the same chain u' , it must be that either $d \leq d'$ or $d' \leq d$. Thus, the decided values agree. Now suppose that $i < j$. By Lemma 6.1, we know that $d \leq c$ for all inputs $\langle j, c \rangle$ to tree turtle j . By Turtle-Unanimity, we have that $d \leq d'$.

SMR-Validity: Suppose that some processor p decides d . This means that p produces an output $\langle i, d, u \rangle$ for some tree turtle i and chain u . By Turtle-Validity, there must exist an input $\langle i, c \rangle$ to the same tree turtle such that $u \leq c$. So, c was proposed by some processors. And since we must have $d \leq u$, we know $d \leq c$.

SMR-Relay: If a correct processor decides d as a result of its output of tree turtle i , then by Lemma 6.1 and Turtle-Unanimity, any processor that completes tree turtle $i+1$ will decide d or an extension of d , and Lemma 6.2 gives us that all correct processors will do exactly as such.

SMR-Monotonicity: If a processor decides d as a result of its output of tree turtle i , then by Lemma 6.1 and Turtle-Unanimity, any processor that completes tree turtle $i+1$ will decide d or an extension of d . So by induction, any later decision will be monotonically extending d . \square

The above protocol does not guarantee SMR-Progress. However, the fact that the correct processors eventually complete each turtle (Lemma 6.2) can be used to make an auxiliary argument for SMR-Progress for a specific protocol (for instance, using a probabilistic termination argument).

7 Tree Turtle Implementations

Here we present two possible tree turtle implementations.

7.1 One-Step Tree Turtle

Our first tree turtle protocol uses only a single round of communication between processors, making it a *one-step* protocol [3]. It requires a quorum system satisfying 3-intersection.

A processor p executing the One-Step Tree Turtle protocol proceeds as follows:

- 1a. p produces an input $\langle c \rangle$ and broadcasts it to all processors, including itself;
- 1b. p waits to receive inputs $\langle c_s \rangle$ from all processors s in any quorum Q_p ;
- 1c. p produces $\langle d, u \rangle$, where:
 - i. $d = \prod_{s \in Q_p} c_s$;
 - ii. Let $C_p = \{ \prod_{s \in Q_p \cap Q} c_s \mid Q \in \mathcal{Q} \}$. Then $u = \max(C_p)$.

That is, d is simply the longest common prefix of the received proposals. To compute u , p considers all quorums Q . For each such quorum, p determines the longest common prefix on the proposals it received from the processors in Q . We show below that these subchains agree with one another.

Lemma 7.1. *All elements of C_p agree.*

Proof. All of the elements in C_p are computed by taking the intersection of Q_p with another quorum. Let Q, Q' be any two quorums and x, x' elements of C_p where $x = \prod_{s \in Q_p \cap Q} c_s$, and $x' = \prod_{s \in Q_p \cap Q'} c_s$. Since Q satisfies 3-intersection, $Q_p \cap Q \cap Q'$ is non-empty. Let r be some processor in this intersection. By the use of \prod , we have that $x \leq c_r$ and $x' \leq c_r$. This means that either $x \leq x'$ or $x' \leq x$. Thus, all elements of C_p agree with each other. \square

Since \leq is transitive, C_p has a maximum element according to \leq , and so the computation of u is well-defined. Further, all elements of C_p are extensions of d :

Lemma 7.2. *For all $x \in C_p$, $d \leq x$.*

Proof. For any quorum Q , we can observe that $Q_p \cap Q \subseteq Q_p$. The longest common prefix over a subset of inputs belonging to processors Q_p is at least as long as the longest common prefix over Q_p . Thus, $d \leq x$ for any $x \in C_p$. \square

We now show that the protocol implements a tree turtle.

Theorem 7.3. *The One-Step Tree Turtle protocol satisfies the tree turtle specification (Definition 5.1).*

Turtle-Termination: Suppose that all correct processors construct an input to the tree turtle. The only point in the protocol where a given correct processor p will wait is to receive messages from a quorum of processors at step 1b. Since there is assumed to be a quorum Q^* that consists entirely of correct processors, and the network reliably delivers messages between correct processors, p will need to wait

no longer than it takes for the messages from all processors in Q^* to be delivered. Thus, p will be able to complete the turtle at step 1c.

Turtle-Agreement: Suppose that two processors p and p' produce $\langle d, u \rangle$ and $\langle d', u' \rangle$, respectively. For all processors $r \in Q_p \cap Q_{p'}$, both p and p' received the proposal c_r from r . Letting $x = \prod_{r \in Q_p \cap Q_{p'}} c_r$, we see that x is present in both C_p and $C_{p'}$. By Lemma 7.2, $d \leq x$, and by the maximality of u' over $C_{p'}$, $x \leq u'$. So, $d \leq u'$ by transitivity. The same argument can be used to show that $d' \leq u$.

Turtle-Unanimity: Suppose that there exists a common prefix $w \leq c$ for all inputs $\langle c \rangle$ to the turtle. Because of step 1b in the protocol, all of the c_s values used to compute $\langle d, u \rangle$ came from inputs $\langle c_s \rangle$. This means that w is a prefix of each c_s , and so w must be a (not necessarily strict) prefix of the longest common prefix $d = \prod_{s \in Q_p} c_s$ of the proposals.

Turtle-Validity: Suppose that p produces an output $\langle d, u \rangle$. We know that $u = \prod_{s \in Q_p \cap Q} c_s$ is a prefix of all proposals made by processors in $Q_p \cap Q$ for some quorum Q . So, taking any processor r in the intersection $Q_p \cap Q$, we know that $u \leq c_r$, where $\langle c_r \rangle$ was the input produced by r . \square

7.2 Lower-Bound Tree Turtle

Now we will present a second tree turtle protocol, the Lower-Bound Tree Turtle. This protocol meets the lower bound on the intersection properties of the quorum system needed to solve SMR: namely, 2-intersection in the crash failure case [2]. With these weaker assumptions about the quorum system, we can design a protocol that makes it safe for processors to output a chain based on messages from all processors in a quorum under the condition that the chains in all such messages agree. Satisfying this condition requires an additional round of communication.

A processor p executing the Lower-Bound Tree Turtle protocol proceeds as follows:

- 1a. p produces an input $\langle c \rangle$ and broadcasts it to all processors, including itself;
- 1b. p waits to receive inputs $\langle c_s \rangle$ from all processors s in any quorum Q_p^1 ;
- 1c. p computes $x = \prod_{s \in Q_p^1} c_s$;
- 2a. p broadcasts $\langle x \rangle$ to all processors, including itself;
- 2b. p waits to receive messages $\langle x_s \rangle$ from all processors s in a quorum Q_p^2 ;
- 2c. p produces $\langle d, u \rangle$, where $d = \min_{s \in Q_p^2} x_s$ and $u = \max_{s \in Q_p^2} x_s$.

First we show that all chains computed in step 1c agree:

Lemma 7.4. *If processors p and p' compute x and x' in step 1c respectively, then $x \simeq x'$.*

Proof. Since there must be some processor r in the intersection $Q_p^1 \cap Q_{p'}^1$, both p and p' received the same chain c_r from

r . Then, since both x and x' are prefixes of c_r , x and x' must agree. \square

Note that the preceding lemma shows that the rules to compute d and u are well-defined since all of the chains x_s for $s \in Q_p^2$ must agree. The next lemma is trivial and shows that a common prefix to all proposals is preserved in the chains computed at step 1c.

Lemma 7.5. *If there exists a chain w such that $w \leq c$ for all inputs $\langle c \rangle$, then for any processor p that computes x in step 1c, $w \leq x$.*

We now show that the Lower-Bound protocol implements the specification of a tree turtle.

Theorem 7.6. *The Lower-Bound Tree Turtle protocol satisfies the tree turtle specification (Definition 5.1).*

Turtle-Agreement: Suppose that two processors p and p' produce $\langle d, u \rangle$ and $\langle d', u' \rangle$, respectively. There must be some processor r in the intersection $Q_p^2 \cap Q_{p'}^2$, and both p and p' received the same chain x_r from r . By Lemma 7.4 and the minimality of d , it must be that $d \leq x_r$. And by the maximality of u' , we have that $x_r \leq u'$. Therefore, $d \leq u'$. The same argument can be used to show that $d' \leq u$.

Turtle-Unanimity: Suppose that there exists a chain w such that $w \leq c$ for all inputs $\langle c \rangle$, and processor p produces output $\langle d, u \rangle$. Lemma 7.5 shows that w will be a prefix of the minimum chain d received by p in step 2b. \square

Turtle-Termination and **Turtle-Validity** are similar to the proof for One-Step Tree Turtle (Section 7.1).

7.3 Message Size

The protocol we discussed uses messages containing chains that represent the entire state machine history. As the size of this history grows, this quickly becomes impractical. However, it is not necessary for a processor to broadcast a chain in subsequent turtles once it has decided that chain following the construction in Section 6.

If processor p decides chain d as a result of its output of a tree turtle, then any other processor p' that outputs $\langle d', u' \rangle$ from the same tree turtle will have $d \leq u'$ by Turtle-Agreement. Thus, p' already knows the contents of the chain d , and p may omit that chain prefix in its proposals to subsequent turtles.

7.4 Heterogeneous Protocols

The simplest way to construct an SMR protocol using the proposed abstractions is to use a single tree turtle protocol, instantiated an unbounded number of times. There are other options, however.

Different tree turtle protocols may have different normal case or worst case performance properties. The Lower-Bound Tree Turtle, when combined with a leader as discussed in

Section 8, has good normal case performance properties, but it relies on synchrony assumptions for liveness. A similar protocol, borrowing ideas from the Ben-Or protocol using randomness [1], can provide termination almost surely but has bad normal case performance. By alternating between the two protocols, we can achieve the best of both worlds.

8 Leaders as an Abstraction

Processors may make different proposals to a tree turtle, preventing them from being able to decide new chains. This issue of contention has been addressed previously in SMR protocols by using a *leader* which drives all processors to use the same proposals. In existing leader-based consensus protocols, the leader lies in the critical path of the protocol: without a functioning leader, no decisions may be made. We show that using tree turtles, leaders can be easily *factored out* so that their only role is to help the protocol towards making decisions.

Leaders can be introduced to an existing tree turtle protocol as follows:

- The leader ℓ for tree turtle i is processor with identifier $i \pmod{|\mathcal{P}|}$, meaning the role of leader rotates through all of the processors.
- The leader ℓ for tree turtle i broadcasts its input c_ℓ to tree turtle i .
- All processors set a timer and wait for the leader's message. If a processor p receives c_ℓ before the timer expires, then it uses c_ℓ as its input to tree turtle i . Otherwise, it proceeds normally.
- The processors double the length of the timer in each tree turtle.

Under synchronous conditions, the leader is able to eliminate contention. The proof requires reasoning about quorums and messages, but these concepts generalize beyond the protocols presented in Section 7.

Lemma 8.1. *Using the above construction and the protocols in Section 7, if there exists an upper bound Δ such that a message sent between two non-crashed processors is delivered and processed within Δ , then there will be an unbounded number of tree turtles where the leader's message is received by all non-crashed processors before their timers expire.*

Proof. Consider any tree turtle i after the point in the execution where the timers have become larger than $\Delta \cdot 2^{|\mathcal{P}|}$. In general, the leader for a tree turtle may not have started executing that tree turtle at the point when other processors begin waiting for its message. Applying Lemma 6.2, consider a processor p that has begun tree turtle $i + 1$. In all of our protocols, p must wait to receive messages from a quorum before completing the protocol. Thus, must be a quorum Q of processors who have begun tree turtle i . Since $Q \cap Q^*$

is non-empty, there must be a tree turtle in the next $|\mathcal{P}|$ instances whose leader is a correct processor in Q . Let j be the first such instance and let ℓ be the leader for tree turtle j .

Since ℓ has already begun tree turtle i , it must catch up at most $|\mathcal{P}|$ tree turtles to reach j . Let t be the timer length for tree turtle j . Since all timer lengths are greater than 2Δ , ℓ will have received messages from a quorum for every tree turtle up to j . A period of Δ is sufficient for other processors to receive ℓ 's input to tree turtle j . However, this does not include the timers that ℓ must set for tree turtles $i, \dots, j - 1$. Since the timer lengths are doubled after each tree turtle, the total time required is $\Delta + (t/2^{|\mathcal{P}|} + t/2^{|\mathcal{P}|-1} \dots + t/2) = \Delta + (1 - 2^{-|\mathcal{P}|})t$. Since $t > \Delta \cdot 2^{|\mathcal{P}|}$, ℓ 's message will be received before the timers expire. Since there are a constant number of turtles in which failures happen, there will be an unbounded number of such turtles. \square

Theorem 8.2. *Using the above construction and the protocols in Section 7, if there exists an upper bound Δ such that a message sent between two non-crashed processors is delivered and processed within Δ , then SMR-Progress is satisfied for the composition of tree turtles (Section 6).*

Proof. According to Lemma 8.1, there will be an unbounded number of tree turtles where the leader's message is received before all timers expire. Turtle-Unanimity provides that the processors who complete the turtle will decide the leader's chain. It follows that correct processors will always eventually be able to decide a new chain. \square

9 Byzantine Fault Tolerance

In this section, we extend tree turtles to make them Byzantine fault-tolerant (BFT). We also modify the two protocols in Section 7 to turn them into BFT-tree turtles.

9.1 Modified System Model

Failures. The failure model will now provide that a faulty processor may not only crash but also behave arbitrarily: it can deviate from the protocol, send arbitrary or contradictory messages, and fail to send messages to some or all processors. Such a processor is referred to as *Byzantine*.

Digital signatures. Because Byzantine processors can make up arbitrary data, our protocols will use cryptographic digital signatures. We use $\text{DSIG}_p(m)$ to denote a digital signature of data object m created by processor p . We assume that no processors can forge the signatures of correct processors, so that if a processor q witnesses both m and $\text{DSIG}_p(m)$, then q knows that if p is correct, then p produced m .

9.2 BFT-Tree Turtles & BFT-SMR

Designing BFT-tree turtles necessitates re-defining both SMR and tree turtles for the BFT setting. As before, processors executing an SMR protocol propose and decide chains. The

BFT-SMR properties are only slight variations of their crash-tolerant counterparts. Now, we only concern ourselves with the decisions made by correct processors.

Definition 9.1 (Byzantine Fault-Tolerant State Machine Replication Specification).

- *BFT-SMR-Agreement*: if a correct processor p decides chain d and a correct processor p' decides chain d' , then $d \simeq d'$ (even if $p = p'$).
- *BFT-SMR-Validity*: if a correct processor decides chain d , then some processor proposed a chain c such that $d \leq c$.
- *BFT-SMR-Relay*: if a correct processor decides chain d , then eventually all correct processors decide d or an extension of d .
- *BFT-SMR-Monotonicity*: if a correct processor decides chain d and later another chain d' , then $d \leq d'$.
- *BFT-SMR-Progress*: if a correct processor proposes a chain that it has not decided before, then that processor eventually decides a chain it has not decided before.

To define BFT-tree turtles, however, we will take a different approach. Rather than only specifying the behavior of correct processors, we will instead strengthen tree turtle inputs and outputs by exclude some arbitrary behavior. We will call use the terms *BFT-inputs* and *BFT-outputs*, respectively, to distinguish them from the earlier definitions.

To allow protocols to strengthen BFT-inputs and BFT-outputs, they will include an additional, abstract piece of data called *evidence*. The implementation of evidence is determined a specific protocol as needed to satisfy the specification for a BFT-tree turtle. While both of our protocols will use digital signatures in evidence, other common techniques for turning crash-tolerant SMR protocols into BFT protocols, such as requiring more quorum intersection, private network links, or using additional rounds of communication, may be used. In fact, a BFT-tree turtle protocol will only define the evidence needed for a BFT-output of the protocol since BFT-inputs are constructed externally from the protocol—it will turn out from our composition of BFT-tree turtles into BFT-SMR that the evidence in a BFT-input will be defined by the previous tree turtle in the execution.

In a BFT-tree turtle, a processor constructs a BFT-input $\langle c, e_c \rangle$ consisting of a chain c and evidence e_c , and produces a BFT-output $\langle d, u, e_{d,u} \rangle$ with a pair of chains d, u and evidence $e_{d,u}$.

Definition 9.2 (Byzantine Fault-Tolerant Tree Turtle Specification).

- *BFT-Turtle-Termination*: if each correct processor constructs a BFT-input, then eventually each correct processor produces a BFT-output.
- *BFT-Turtle-Agreement*: for any two BFT-outputs $\langle d, u, e_{d,u} \rangle$ and $\langle d', u', e_{d',u'} \rangle$, $d \leq u'$ and $d' \leq u$.
- *BFT-Turtle-Unanimity*: for any chain w , if $w \leq c$ for all BFT-inputs $\langle c, e_c \rangle$, then $w \leq u$ for all BFT-outputs $\langle d, u, e_{d,u} \rangle$.

- *BFT-Turtle-Validity*: if some processor produces a BFT-output $\langle d, u, e_{d,u} \rangle$, then some processor must have produced a BFT-input $\langle c, e_c \rangle$ such that $u \leq c$.

Note that BFT-Turtle-Unanimity is weaker than its crash-tolerant counterpart (Definition 5.1). This weaker property is easier to satisfy in the BFT case and is sufficient for implementing the safety properties of BFT-SMR, although it means that BFT-SMR-Monotonicity is no longer guaranteed.

Evidence is not intended to prevent all Byzantine behavior. Depending on the BFT-tree turtle protocol, faulty processors may be able to produce multiple BFT-inputs or BFT-outputs or use arbitrary chains to produce a BFT-output the protocol. However, evidence allows us to reason about BFT-inputs and BFT-outputs made by correct and Byzantine processors alike to the extent allowed by the constraints defined by the protocol.

9.3 BFT-Tree Turtles Implement BFT-SMR

The composition of BFT-tree turtles is very similar to that of crash tolerant tree turtles (Section 6). As before, all of the processors in \mathcal{P} will execute an unbounded sequence of tree turtles in succession. If a processor completes BFT-tree turtle i and produces a BFT-output $\langle i, d, u, e_{d,u} \rangle$, then that processor decides the chain d *only if* d is longer than its longest decided chain. This is to ensure BFT-SMR-Monotonicity is satisfied. The processor then selects a new chain c to propose satisfying $u \leq c$. It then constructs a BFT-input to the next BFT-tree turtle $i + 1$ as $\langle i + 1, c, \langle i, d, u, e_{d,u} \rangle \rangle$, using its BFT-output from the previous tree turtle as the evidence in its BFT-input.

Lemma 9.3. *If a processor produces BFT-output $\langle i, d, u, e_{d,u} \rangle$ for BFT-tree turtle i , then for all BFT-inputs $\langle j, c, e_c \rangle$ to subsequent BFT-tree turtles $j > i$, it must be that $d \leq c$.*

Proof. The base case is analogous to that given for Lemma 6.1. In the inductive case, if a processor p constructs a BFT-input $\langle j, c, e_c \rangle$ to BFT-tree turtle j , then p must have produced a BFT-output $\langle j - 1, d', u', e_{d',u'} \rangle$ in the previous tree turtle which satisfies $u' \leq c$. The inductive hypothesis gives us that $d \leq c'$ for all BFT-inputs $\langle j - 1, c', e_{c'} \rangle$ to BFT-tree turtle $j - 1$. So the condition for BFT-Turtle-Unanimity is satisfied and this gives us that $d \leq u' \leq c$. \square

Lemma 9.4. *Every correct processor eventually produces a BFT-output for each BFT-tree turtle.*

Proof. The proof is the same as that given for Lemma 6.2, except that the reasoning applies to all *correct* processors rather than all non-crashed processors. \square

Theorem 9.5. *The composition of BFT-tree turtles implements a protocol satisfying BFT-SMR-Agreement, BFT-SMR-Validity, and BFT-SMR-Monotonicity.*

BFT-SMR-Agreement: Suppose that two correct processors p and p' decide chains based on their BFT-outputs $\langle i, d, u, e_{d,u} \rangle$ and $\langle j, d', u', e_{d',u'} \rangle$, respectively. First, suppose that $i = j$. By BFT-Turtle-Agreement, $d \leq u'$ and $d' \leq u$, and thus $d \leq d'$ or $d' \leq d$. Thus, the decided values agree. Now suppose that $i < j$. By Lemma 6.1, we know that $d \leq c$ for all BFT-inputs $\langle j, c, e_c \rangle$ to BFT-tree turtle j . By BFT-Turtle-Unanimity, we have again that $d \leq u'$, so it follows that d and d' agree.

BFT-SMR-Validity: Suppose that a correct processor p decides the chain d . This means that p produces a BFT-output $\langle i, d, u, e_{d,u} \rangle$ for some BFT-tree turtle i , chain u , and evidence $e_{d,u}$. By BFT-Turtle-Validity, there must exist a BFT-input $\langle i, c, e_c \rangle$ to the same tree turtle such that $u \leq c$. So, c was proposed by some processor. And since we must have $d \leq u$, we know $d \leq c$.

BFT-SMR-Monotonicity: If a correct processor decides d as a result of its BFT-output of BFT-tree turtle i , then by Lemma 6.1 and BFT-Turtle-Unanimity, an inductive argument shows that any BFT-output that the processor produces for a later BFT-tree turtle will have a chain d' such that $d \leq d'$ or $d' \leq d$. Since correct processors will only decide a new chain if it is an extension of its longest decided chain, it follows that the processor's decisions are extended monotonically. \square

9.4 BFT One-Step Tree Turtle

The One-Step Tree Turtle (Section 7.1) can be made into a BFT-tree turtle with the following modifications:

- The quorum system Q must satisfy 5-intersection instead of 3-intersection (meeting the lower bound for quorum intersection of BFT one-step protocols [13]);
- In step 1a., processor p broadcasts $\langle \langle c, e_c \rangle, \text{DSIG}_p(c) \rangle$, i.e., its BFT-input and a digital signature for c ;
- In step 1b., processor p must ensure that the received messages contain valid digital signatures and BFT-inputs $\langle c_s, e_{c_s} \rangle$, and for any that do not, p discards the message;
- In step 1c., processor p constructs the evidence $e_{d,u}$ as set of tuples $\{ \langle c_s, \text{DSIG}_s(c_s) \rangle \mid s \in Q_p \}$ and includes it in its BFT-output $\langle d, u, e_{d,u} \rangle$;
- A BFT-output $\langle d, u, e_{d,u} \rangle$ must contain evidence $e_{d,u} = \{ \langle c_s, \text{DSIG}_s(c_s) \rangle \mid s \in Q \}$ for some quorum Q such that the chains c_s allow d and u to be recomputed according to the protocol at step 1c.

The proof of correctness for the BFT One-Step Tree Turtle follows analogous reasoning to that given for the crash tolerant version.

Lemma 9.6. *All elements of C_p agree.*

Proof. All of the elements in C_p are computed by taking the intersection of Q_p with two other quorums. Suppose x, x'

are elements of C_p , where $x = \prod_{s \in Q_p \cap Q_1 \cap Q_2} c_s$, and $x' = \prod_{s \in Q_p \cap Q'_1 \cap Q'_2} c_s$, for some quorums Q_1, Q_2, Q'_1, Q'_2 . Since the quorum system satisfies 5-intersection, $Q_p \cap Q_1 \cap Q_2 \cap Q'_1 \cap Q'_2$ is non-empty. Let r be some processor in this intersection. By the use of \prod , we have that $x \leq c_r$ and $x' \leq c_r$, where c_r is the chain from r used to complete the protocol. This means that either $x \leq x'$ or $x' \leq x$. \square

Lemma 9.7. *For all $x \in C_p$, $d \leq x$.*

Proof. Observe that $Q_p \cap Q_1 \cap Q_2 \subseteq Q_p$. The longest common prefix over a subset of Q_p is at least as long as the longest common prefix over Q_p . Thus, $d \leq x$ for any $x \in C_p$. \square

Theorem 9.8. *The BFT One-Step Tree Turtle protocol satisfies the BFT-tree turtle specification (Definition 9.2).*

BFT-Turtle-Termination: Suppose that all correct processors construct a BFT-input for the protocol. Consider any correct processor p . By construction of the protocol, p will wait to receive messages from a quorum of processors at step 1b. This is the only point in the protocol where p will wait. Since there is at least one quorum Q^* that consists entirely of correct processors, and the network reliably delivers messages between correct processors, p will eventually receive messages from all processors in a quorum. Furthermore, all messages sent by processors in Q^* will contain BFT-inputs and valid digital signatures, as correct processors do not send bad messages in step 1a. Thus, p will not discard any of these messages and will be able to complete the turtle at step 1c.

BFT-Turtle-Agreement: Suppose that two processors p and p' produce BFT-outputs $\langle d, u, e_{d,u} \rangle$ and $\langle d', u', e_{d',u'} \rangle$, respectively. Let Q and Q' be the quorums used to construct $e_{d,u}$ and $e_{d',u'}$, respectively. For all (correct) processors $r \in Q \cap Q' \cap Q^*$, both e and e' must contain the same digitally signed chain which was sent by r : $\langle c_r, \text{DSIG}_r(c_r) \rangle$. This is because correct processors do not send different messages to different processors at step 1a., so r must have sent the same message to both p and p' . Furthermore, the digital signatures of correct processors cannot be forged, so p and p' must have used the signature that r actually constructed in step 1a. Thus, if the chains in e are used in step 1c. to compute $x = \prod_{s \in Q \cap Q' \cap Q^*} c_s$ and the chains in e' are used to compute $x' = \prod_{s \in Q \cap Q' \cap Q^*} c_s$, we would have that $x = x'$. Thus p would have $x \in C_p$ and p' would have $x \in C_{p'}$. By Lemma 7.2, if e is used to compute d , we would have that $d \leq x$. And, if e' is used to compute d' , it must be that $x \leq d'$. Therefore $d \leq d'$, and an analogous argument shows that $d' \leq d$.

BFT-Turtle-Unanimity: Assume that $w \leq c$ for all BFT-inputs $\langle c, e_c \rangle$ to the protocol. For any BFT-output $\langle d, u, e_{d,u} \rangle$, let Q be the quorum used to construct $e_{d,u}$. Then for all $r \in Q \cap Q^*$, the digitally signed chain $\langle c_r, \text{DSIG}_r(c_r) \rangle$ in e

must have been produced by the correct processor r . Because of step 1a., correct processors only sign chains c_r that are part of BFT-inputs $\langle c_r, e_{c_r} \rangle$, meaning that $w \leq c_r$. Thus, taking any third quorum Q' , if e is used to compute $x = \prod_{s \in Q \cap Q' \cap Q^*} c_s$, it follows that $w \leq x$. Therefore, by Lemma 9.6, it must be that $x \leq u$ which implies $w \leq u$.

BFT-Turtle-Validity: If a processor produces a BFT-output $\langle d, u, e_{d,u} \rangle$, then by quorum intersection it must be that $u \leq c_r$ for some correct processor $r \in Q^*$, where $\langle c_r, e_{c_r} \rangle$ is the BFT-input constructed by r . \square

9.5 BFT Lower-Bound Tree Turtle

The Lower-Bound Tree Turtle (Section 7.2) can be made into a BFT-tree turtle with the following modifications:

- The quorum system Q must satisfy 3-intersection;
- In step 1a., processor p broadcasts $\langle \langle c, e_c \rangle, \text{DSIG}_p(c) \rangle$, i.e., its BFT-input and a digital signature for c ;
- In step 1b., processor p must ensure that the received messages contain valid digital signatures and BFT-inputs $\langle c_s, e_{c_s} \rangle$, and for any that do not, p discards the message;
- In step 2a., processor p broadcasts $\langle x, \text{DSIG}_p(x), \{ \langle c_s, \text{DSIG}_s(c_s) \rangle \mid s \in Q_p^1 \} \rangle$, i.e., the computed chain x , a digital signature for x , and all of the chains and digital signatures that it used to complete step 1c. of the protocol;
- In step 2b., processor p must ensure that the received messages contain valid digitally signed chains such that x_s can be recomputed according to the protocol at step 1c., and for any that do not, p discards the message;
- In step 2c., processor p constructs the evidence $e_{d,u}$ as set of tuples $\{ \langle x_s, \text{DSIG}_s(x_s) \rangle \mid s \in Q_p^2 \}$ and includes it in its BFT-output $\langle d, u, e_{d,u} \rangle$;
- A BFT-output $\langle d, u, e_{d,u} \rangle$ must contain evidence $e_{d,u} = \{ \langle x_s, \text{DSIG}_s(c_s) \rangle \mid s \in Q \}$ for some quorum Q such that the chains c_s allow d and u to be recomputed according to the protocol at step 2c. and all of the chains x_s agree.

The BFT Lower-Bound Tree Turtle requires an extra condition on the digitally signed chains included in evidence for BFT-outputs which is not necessary for the BFT One-Step Tree Turtle. This ensures that the minimum and maximum of those chains is always well-defined. Further, this property is always satisfied by the evidence constructed by a correct processor executing the protocol due to the following:

Lemma 9.9. *If processor p broadcasts a message $m = \langle x, \text{DSIG}_p(x), \{ \langle c_s, \text{DSIG}_s(c_s) \rangle \mid s \in Q \} \rangle$ and p' broadcasts a message $m' = \langle x', \text{DSIG}_{p'}(x'), \{ \langle c_s, \text{DSIG}_s(c_s) \rangle \mid s \in Q' \} \rangle$ at step 2a. such that the digitally signed chains included in m and*

m' allow x and x' to be recomputed, respectively, according to the protocol at step 1c., then x and x' agree.

Proof. For any processor r in the intersection $Q \cap Q' \cap Q^*$, both m and m' will contain the same digitally signed chain $\langle c_r, \text{DSIG}_r(c_r) \rangle$ from r . This is because correct processors only send a single message to all processors at step 1b. of the protocol. Thus both x and x' must be prefixes of c_r and therefore agree with each other. \square

Lemma 9.10. *For any chain w such that $w \leq c$ for all BFT-inputs $\langle c, e_c \rangle$, then if processor p that broadcasts a message $m = \langle x, \text{DSIG}_p(x), \{ \langle c_s, \text{DSIG}_s(c_s) \rangle \mid s \in Q \} \rangle$ at step 2a., such that the digitally signed chains included in m allow x to be recomputed according to the protocol at step 1c., then w and x agree. Further, if p is correct, then $w \leq x$.*

Proof. Since Q must contain at least one correct processor r due to quorum intersection, m contains a digitally signed chain $\langle c_r, \text{DSIG}_r(c_r) \rangle$ such that $w \leq c_r$. This is because correct processors will only produce such a signature in step 1a. for chains c_r that are part of BFT-inputs $\langle c_r, e_{c_r} \rangle$. Therefore $x \leq c_r$ and so x and w agree.

If p is correct, then p will only use messages containing BFT-inputs to compute x at step 1c. of the protocol. By assumption, w is a prefix of the chains contained in all such inputs. \square

Theorem 9.11. *The BFT Lower-Bound Tree Turtle protocol satisfies the BFT-tree turtle specification (Definition 9.2).*

BFT-Turtle-Agreement: Suppose that two processors p and p' produce BFT-outputs $\langle d, u, e_{d,u} \rangle$ and $\langle d', u', e_{d',u'} \rangle$, respectively. Let Q and Q' be the quorums over which the evidence $e_{d,u}$ and $e_{d',u'}$ are constructed, respectively. Then, there must be a correct processor r in the intersection $Q \cap Q'$, and both e and e' must include the chain x_r broadcast by r in step 2b. By the minimality of d , it must be that $d \leq x_r$. And by the maximality of u' , we have that $x_r \leq u'$. Therefore, $d \leq u'$, and an analogous argument shows $d' \leq u$.

BFT-Turtle-Unanimity: Suppose that $w \leq c$ for all BFT-inputs $\langle c, e_c \rangle$, and that processor p produces BFT-output $\langle d, u, e_{d,u} \rangle$. Letting Q be the quorum over which $e_{d,u}$ is constructed, quorum intersection ensures that there is at least one correct processor r in Q . By Lemma 7.5, if r broadcasts a message including the chain x_r in step 2b, then $w \leq x_r$. Further, x_r must be equal to, or a prefix of, the maximum chain in $e_{d,u}$. Thus, $w \leq x_r \leq u$.

Turtle-Termination & Turtle-Validity: the proofs are analogous to those given for Theorem 9.8. \square

10 Related Work

Abstracting the problem of achieving consensus on a single value to entire sequences was previously applied to Paxos

in Generalized Paxos [10]. This work further generalizes chains to partial-orders of values in which non-interfering values commute. The HotStuff protocol [14] conceptualizes the state machine history as a tree, but it only extends the tree by a single node at a time. The same can be said of blockchain protocols.

The idea of heterogeneous SMR protocols is similar to protocols which have different “modes”. Typically there is one mode which is considered the normal operation of the protocol and another designed for fast-tracking decisions under best-case conditions. One such example is Fast Paxos [11] which is able to skip a round of communication in periods without contention. These modes, however, are usually considered to be part of a single protocol instead of separate protocols satisfying a common specification.

11 Conclusion & Future Work

SMR protocols have been around for over thirty years. We revisit the structure of these protocols and propose new abstractions—*trees and turtles*—for the design of modular SMR protocols.

While this paper did not discuss the performance of tree turtle protocols, we believe that they have potential to be performant through their ability to drive long extensions to the state machine history in a few rounds of communication. Future work could include an empirical analysis of their performance. Further techniques for optimization can also be investigated such as pipelining chains from different rounds of the protocol simultaneously.

There is work in progress by the authors on building formally verified consensus and SMR protocols in both Dafny and Coq based on the presented abstractions due to their simplicity and efficiency. We also expect our abstractions to support different flavors of SMR, such as ordered consensus and heterogeneous consensus, by utilizing the additional structure of trees and the flexibility of the tree turtle composition. Further generalizations of trees and chains into more general algebraic structures are also possible.

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Appendix A: Threshold Quorum Systems

For completeness, we include how a k -intersecting quorum system can be implemented (based on [7]). A *threshold quorum system* constructs quorums from all subsets of \mathcal{P} above a particular size. Let $n = |\mathcal{P}|$ be the number of processors. Let f be the maximum number of processors that may be faulty. We can construct a k -intersecting quorum system for

$k \geq 1$ such that $n > k \cdot f$ as follows: the quorums are all subsets of \mathcal{P} that have at least $n - f$ processors. Then Q^* is the quorum that contains the $n - f$ correct processors. In a k -intersecting quorum system implemented with a threshold quorum system, increasing the value for k results in an increase in the number of correct processors that are assumed to exist in the system.